A tank is being used to store gas. The pressure inside the tank changes as the gas changes temperature. **SCORE: ______ / 3 PTS**

When the temperature of the gas is $t^\circ$ Celsius, the pressure increases by $p(t)$ Pascals per degree Celsius that the temperature rises.

What is the meaning of the equation $\int_{55}^{70} p(t)\, dt = 40$ in this situation ?

**NOTES:** **Your answer must use all three numbers from the equation, along with correct units.**
**Your answer should NOT use "*t*", "*p*(*t*)", "integral", "antiderivative", "rate of change" or "derivative".**

THE PRESSURE IN THE TANK INCREASES 40 PASCALS
IF THE TEMPERATURE OF THE GAS RISES FROM 55°C TO 70°C

If $k(x) = \int_{-2x}^{x^4} \sqrt{1+t^3}\, dt$, find $k'(-1)$. **SCORE: ______ / 4 PTS**

$$k'(x) = (x^4)'\sqrt{1+(x^4)^3} - (-2x)'\sqrt{1+(-2x)^3}$$

$$= \underbrace{4x^3}_{\frac{1}{2}}\underbrace{\sqrt{1+x^{12}}}_{1} + \underbrace{2}_{\frac{1}{2}}\underbrace{\sqrt{1-8x^3}}_{1}$$

$$k'(-1) = -4\sqrt{2} + 2\sqrt{9}$$

$$= \underbrace{6-4\sqrt{2}}_{1}$$

Answer the following questions about the definition of the definite integral as presented in lecture. **SCORE: ______ / 3 PTS**
(Your answers may refer to the fact that the definite integral equals the area under a curve which is above the $x$ – axis.)

[a] Why is there a summation in the definition ?

THE AREA IS APPROXIMATED BY THE SUM OF AREAS OF RECTANGLES

[b] What does the inequality regarding $x_i^*$ $(a+(i-1)\Delta x \le x_i^* \le a+i\Delta x)$ mean ? Recall that $i$ goes from $1$ to n.

ONE X-VALUE IS TAKEN FROM EACH SUBINTERVAL TO CREATE A RECTANGLE (WITH HEIGHT EQUAL TO THE CORRESPONDING f VALUE)

Evaluate the following integrals. **SCORE: ______ /10 PTS**

SUBTOTAL = 4 POINTS

[a] $\displaystyle\int \frac{(3-x)^2}{\sqrt{x}}\,dx$

$= \displaystyle\int \frac{9-6x+x^2}{x^{\frac{1}{2}}}\,dx$

$= \underbrace{\displaystyle\int \left(9x^{-\frac{1}{2}} - 6x^{\frac{1}{2}} + x^{\frac{3}{2}}\right)dx}_{1}$

$= 9\cdot 2x^{\frac{1}{2}} - 6\cdot\frac{2}{3}x^{\frac{3}{2}} + \frac{2}{5}x^{\frac{5}{2}} + C$

$= \underbrace{18x^{\frac{1}{2}}}_{1}\ \underbrace{-4x^{\frac{3}{2}}}_{1} + \underbrace{\frac{2}{5}x^{\frac{5}{2}}}_{\frac{1}{2}} \underbrace{+C}_{\frac{1}{2}}$

SUBTOTAL = 4 POINTS

[b] $\displaystyle\int_{-1}^{1} \frac{5x^2}{6-2x^3}\,dx$ ← NOT ODD

$\frac{1}{2}$ $\underline{u = 6-2x^3}$ $\begin{cases} x=1 \longrightarrow u=4 \\ x=-1 \longrightarrow u=8 \end{cases}$

$\frac{du}{dx} = -6x^2$

$dx = \frac{-du}{6x^2}$

$\frac{5x^2}{6-2x^3}\,dx = \frac{5x^2}{6-2x^3}\cdot\frac{-du}{6x^2}$

$= -\frac{5}{6}\frac{1}{u}\,du$

$\displaystyle\int_8^4 \underbrace{-\frac{5}{6}\frac{1}{u}\,du}_{1} = \underbrace{-\frac{5}{6}\ln|u|}_{1}\Big|_8^4$

$= \underbrace{-\frac{5}{6}(\ln 4 - \ln 8)}_{1}$

EITHER ONE OK $\frac{1}{2}$ $= \underline{-\frac{5}{6}\ln\frac{1}{2} = \frac{5}{6}\ln 2}$

SUBTOTAL = 2 POINTS

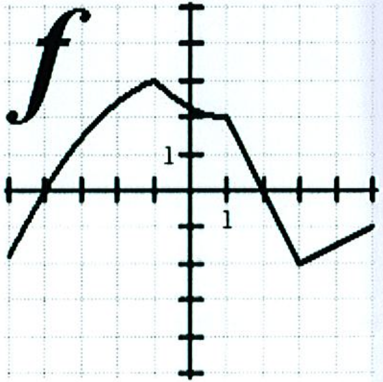
[c] $\displaystyle\int_{-\pi}^{\pi} \sin^3 x\cos^5 x\,dx$

1 $\boxed{\begin{aligned} &\sin^3(-x)\cos^5(-x) = (-\sin x)^3\cos^5 x \\ &= -\sin^3 x\cos^5 x \end{aligned}}$

INTEGRAND IS ODD + $\underbrace{\text{CONTINUOUS}}_{\frac{1}{2}}$

INTEGRAL $\underline{= 0}$ $\frac{1}{2}$

Let $g(x) = \int_{1}^{x} f(t)\, dt$, where $f$ is the function whose graph is shown on the right. **SCORE: ______ / 6 PTS**




[a] Find $g'(-1)$. **Explain your answer very briefly.**

1 POINT EACH

$g'(-1) = f(-1) = 3$

[b] Find all intervals over which $g$ is concave down. **Explain your answer very briefly.**

$g'(x) = f(x)$ IS DECREASING ON $(-1, 3)$

[c] Find the $x$ – coordinates of all local minima of $g$. **Explain your answer very briefly.**

$g'(x) = f(x)$ CHANGES FROM NEGATIVE TO POSITIVE AT $x = -4$

**In complete sentences, using proper English and mathematical notation,** state the Fundamental Theorem of Calculus (both parts) and the Net Change Theorem.

SCORE: ______ / 4 PTS

IF $f$ IS CONTINUOUS ON $[a,b]$

① IF $g(x)=\int_a^x f(t)\,dt$, THEN $g'(x)=f(x)$ ON $(a,b)$

② IF $F'(x)=f(x)$, THEN $\int_a^b f(x)\,dx=F(b)-F(a)$

IF $F'$ IS CONTINUOUS ON $[a,b]$, THEN $\int_a^b F'(x)\,dx=F(b)-F(a)$